The Narrow-V Wake

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***Abstract*—The narrow-V ship wake in a radar image is typically a bright V-shaped or wedge-like return with an angle of a few degrees. It is often seen in images from satellite-borne Synthetic Aperture Radar (SAR), especially when the ship is traveling in a direction close to the SAR track and when the wind is light. Though explanation for the narrow-V wake exists already, it has been recognized from the time of Seasat that the model is deficient at microwave frequencies higher than L-band. The purpose of this paper is to describe a new model for the narrow-V wake based on the theory of unsteady moving sources. It overcomes the deficiency and implies that the narrow-V wake could be useful for ship classification purposes.**

***Index Terms*---Ship wake, gravity waves, ship wake angle, ship wake length. SAR, RADARSAT-2**

I. Introduction

NARROW-V SHIP wakes have been observed in Synthetic Aperture Radar (SAR) images from Seasat, which operated at L-band in HH polarization, through ERS-1 and 2, which operated in VV-polarization at C-band to TerraSARX, which carries X-band radar. The wake is often observed when the ship is traveling close to the azimuthal direction, which is parallel to the satellite track; the wind speed is usually less than a few meters per second. An example of a narrow-V wake extracted from a RADARSAT-2 fine beam image of the Georgia Strait is shown in Fig. 1.

In this image, which is a combination of VV and VH polarizations, the returns from the ship are so strong that azimuth and range sidelobes are prominent. The Westwood Rainier is a cargo ship of length 200 m, beam 31 m and draft 11.5 m. The ship speed was 8.5 m/s and its bearing was almost opposite to radar azimuth. The wind was blowing from the starboard at about 3.6 m/s. The wake extended for about 6 km and the angle between the two arms was about 5°.

An understanding of the narrow-V wake is important to determine if information about the ship can be extracted. Such information might be useful for classification purposes and to validate data from other sources. The purpose of the present work is to examine previous wake models and to introduce a new one that overcomes a basic deficiency. The focus of the new model is the wake created by the ship's propeller though it is applicable to other unsteady sources, such as those associated with ship motions and the reflection of ambient waves.

There have been several attempts to determine the physical mechanisms that lead to the formation of narrow-V wakes. For example, Case et al. [2] investigated the wakes in Seasat images. They studied a model based on the Kelvin wake and another based on the propagation of waves at the Bragg wavelength produced by the ship.

The Kelvin wake was a candidate because the wave pattern, which is stationary in the ship's reference frame, includes divergent waves, which are short wavelength waves some of which propagate nearly perpendicularly to the ship's track. It was considered that short wavelength waves might propagate with a group velocity that was sufficiently small to produce a pair of linear returns at typical narrow-V angles.

To produce a large coherent radar return, the wave vector of the waves should be equal to the Bragg wave vector. That is, it should be in the direction (or opposite to) the range direction of the radar and have the appropriate Bragg wavelength; effectively there are two conditions for each arm.

The problem with this model is that it is difficult to satisfy the conditions reliably for one arm and especially to satisfy them simultaneously for the two arms. Therefore the pure Kelvin wake model was discarded.

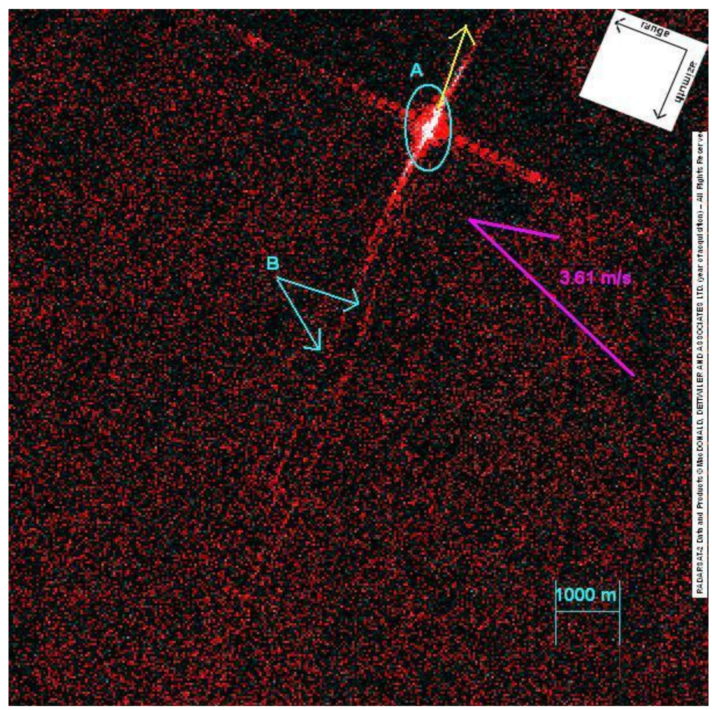

Fig. 1. Image of the Westwood Ranier and its narrow-V wake. A: ship return. B: narrow-V wake arms. (From Roy [1].)

The next approach was to relax the stationarity condition in the Kelvin wake by assuming a randomization of the phases of short wavelength waves generated close to the ship. Now, the waves at a point in the wake can exist with a variety of crest orientations and all that is required is to match the waves to the Bragg wavelength. This is known as the Interrupted Kelvin Wake

(IKW) model and it has often been invoked [3,4]. Case et al. [2] consider that the source of the waves could be turbulence in the boundary layer, breaking waves near to the ship or complicated turbulent flows from the propeller.

Using the IKW, Stapleton [5] has successfully explained narrow-V wakes at L-band and P-band. Reed and Milgram [6] essentially refer to the IKW for L-band but Hasselwimmer [7] does not mention radar frequency dependence.

Balser et al. [8] have observed narrow-V wakes that appear to confirm the IKW model. They used shore-based coherent radar to measure the positions and Doppler velocities of ocean features at L-band and X-band. The Doppler spectra indicated that the returns from the wake arms were due to Bragg scattering and that the wake angles were more or less consistent with the ship speed combined with the group velocity of Bragg waves; the group velocities at L-band and X-band happened to be similar.

However, as pointed out by Case et al. [2], waves propagating at the Bragg wavelengths appropriate to microwave radar tend to be damped. Though the IKW might be an appropriate model at L-band with high incidence angles, it cannot explain narrow-V wakes at C-band and X-band.

The unsteady wake as described by Lighthill [10] provides the basis for the new model. Strong attenuation is avoided because the waves are required neither to propagate in a specific direction nor to have the Bragg wavelength. If accepted, the model implies that the Bragg scattering occurs as a result of second order processes, such as hydrodynamic modulation, breaking waves or the interaction of the unsteady wake with ambient waves.

II. Theory

Case et al. [2] point out that surface gravity waves are damped by viscosity. The rate of loss of energy, E, is given by:

$$\frac{1}{E}\frac{dE}{dt} = -4\nu k^2 \tag{1}$$

where ν is the kinematic viscosity of water, which is about 10^{-6} m^2/s, and k is the angular wave number. The Bragg wavelength, λ_B, depends on the radar wavelength, λ, and the angle of incidence, i:

$$\lambda_B = \frac{\lambda}{2\sin i} \tag{2}$$

At high angles of incidence, as used by Balser et al. [8], the L-band Bragg wavelength was about 10 cm. For Seasat, with an angle of incidence of about 20° it was about 34 cm.

Lighthill [10] shows that equation (1) is valid even when surface tension is taken into account in the propagation of short capillary waves. From equation (1), a characteristic attenuation time (often called the e-folding time) can be expressed as a function of wavelength. This is plotted in Fig. 2. It can be seen that, if the Bragg wavelength is 30 cm, the wave can persist for about 600 s. If the Bragg wavelength is reduced to 10 cm, as in Balser et al., the wave only persists for about 60 s. As implied by Case et al. [2], their model is somewhat marginal because Seasat wakes are sometimes visible for much longer than 600 s. At X-band and C-band, using typical SAR angles of incidence, the waves persist only for a few seconds and the model fails. Because Balser et al. [8] present their data for just a few minutes, their L-band observations support the IKW; their X-band results are actually inconclusive.

Gravity wave wakes can be excited by time varying sources moving with the ship and again create a wake pattern but, unlike the Kelvin wake, this is not stationary in the ship's reference frame [10]. For sinusoidal sources there are distinct wake crest patterns but here a more general approach will be developed based on a simple propagation model as is done for the IKW. It is assumed that the ship carries a pulsating source with angular frequency, ω_0 (such as a propeller). The pulsations are Doppler shifted in the reference frame of the ocean:

$$\omega_0 = \omega - kU\cos\theta \tag{3}$$

where ω is the angular frequency of the waves in the ocean frame, θ is the angle between the propagation vector and the ship's track and U is the ship speed. This can be combined with the dispersion relation for gravity waves in deep water:

$$\omega^2 = gk \tag{4}$$

where ω is the angular frequency of the wave and g is the acceleration due to gravity. Because we are not for the moment interested in very short capillary waves that will be strongly attenuated, surface tension is ignored.

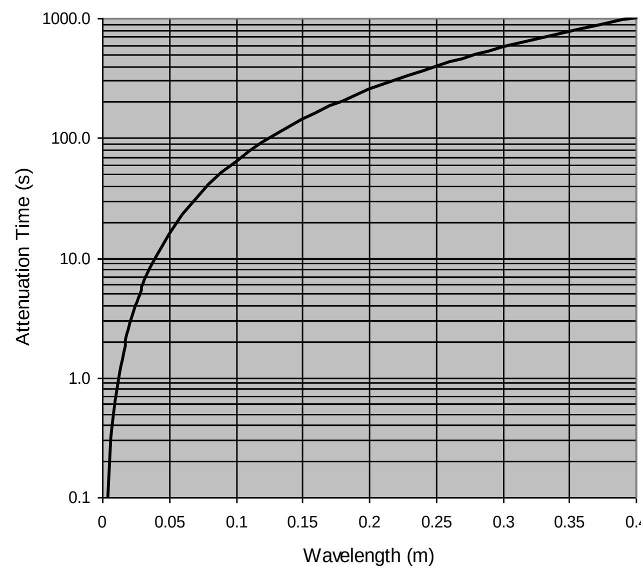

Fig. 2. The attenuation time due to viscous damping as a function of wavelength.

It is useful to introduce a set of dimensionless variables:

$$\begin{aligned} \Omega &= \omega U / g \\ \kappa &= kU^2 / g \\ C_g &= c_g / U = U^{-1} d\omega / dk \end{aligned} \tag{5}$$

where c_g is the group velocity. Equations (3) and (4) become:

$$\begin{aligned} \Omega_0 &= \Omega - \kappa\cos\theta \\ \Omega^2 &= \kappa \end{aligned} \tag{6}$$

Combining equations (6) to eliminate Ω, we have:

$$\kappa = \frac{1 - 2\Omega_0\cos\theta \pm \sqrt{1 - 4\Omega_0\cos\theta}}{2\cos^2\theta} \tag{7}$$

This yields the normalized wave number in terms of the excitation frequency in the reference frame of the ship and the angle that the wave vector makes with the ship track. It is important to note that the argument of the square root can be negative for $|\Omega_0| > 0.25$, which indicates that some wave vectors are forbidden. Also positive and negative frequencies must be considered; a simple oscillating source is represented by a power spectral density that is symmetric across the spectrum axis. There are four solutions altogether because there are two possibilities for the sign of the square root. However, just two of them are independent; the other two are invalid [9]. As shown in [9,10], the two solutions correspond to two separate wake patterns associated with positive and negative frequency components.

Therefore, for any angle θ in one of the patterns, there is usually only one value of κ (see [9]).

The initial disturbance created by the ship travels a distance determined by the group velocity. From geometrical considerations [2], the wake half angle, β, formed by waves at angle θ to the track is given by:

$$\tan\beta = \frac{c_g \sin\theta}{U - c_g \cos\theta} = \frac{C_g \sin\theta}{1 - C_g \cos\theta} \quad (8)$$

where it easy to show that $C_g = 1/(2\kappa^{1/2})$. For each value of θ, a value of β can be calculated provided a solution exists.

It turns out that the wake half angle, β, exhibits a maximum and this is plotted out for high values of Ω_0 in Fig. 3. There are two curves corresponding to the two wake solutions. These are similar to those presented by Lighthill [10] over the common range of Ω_0 between 1 and 2.5. As in the Kelvin wake, the maximum angle corresponds to a caustic where divergent and transverse waves coalesce; amplitudes are likely to be high on lines of cusp waves. It is worth noting that the angle between the cusp wave vector and the ship's track appears to approach 125° asymptotically at large Ω_0; it is about 100° for positive frequencies when $\Omega_0 = 1$.

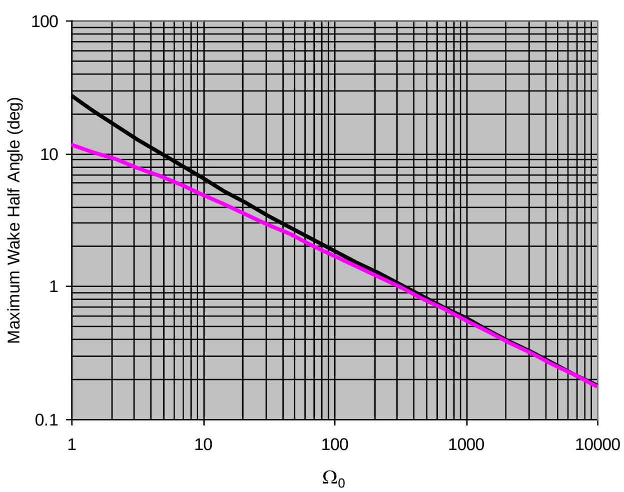

Fig. 3. The wake half angle as a function of Ω_0 for positive frequencies (—) and negative frequencies (—).

The upper curve is approximately a straight line. In the range of Ω_0 depicted, the wake half angle is given (in degrees) by:

$$\beta \approx 19\,\Omega_0^{-1/2} \quad (9)$$

It is important to verify that the cusp waves are not heavily attenuated. Fig. 4 shows the normalized group velocity for the cusp waves. The calculation is based on the assumption that the cusp waves are not influenced by surface tension. Once again the upper curve is approximately a straight line, which can be represented as:

$$C_g \approx 0.37\,\Omega_0^{-1/2} \quad (10)$$

To enable the attenuation to be determined easily, Fig. 5 shows a graph of the e-folding distance as a function of un-normalized group velocity. The attenuation distance is calculated by multiplying the e-folding time by the group velocity. The group velocity calculation employs the dispersion relation for very short capillary waves [10] with a value of surface tension over water density equal to $7.4\text{x}10^{-5}$ $m^3.s^{-2}$.

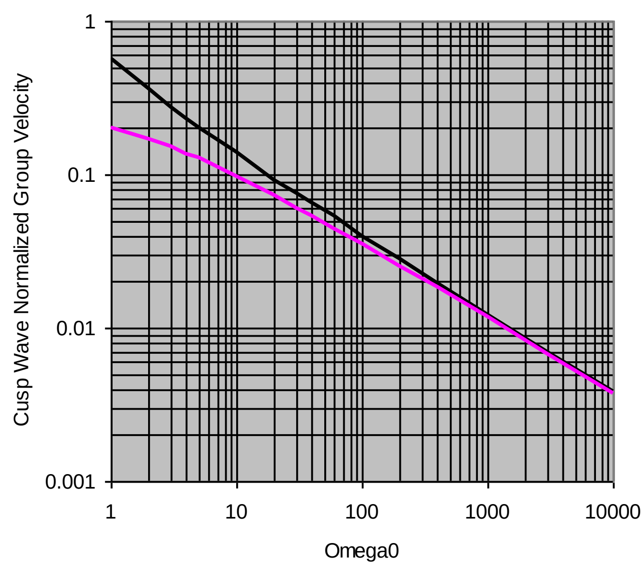

Fig. 4. The normalized group velocity of cusp waves as a function of Ω_0 for positive frequencies (—) and negative frequencies (—).

III. Applications

Consider a ship moving at 8.5 m/s with a four bladed propeller rotating at 125 rpm. These are reasonable parameters for the class of cargo ship imaged in Fig. 1. The blade frequency of the propeller is 8.3 Hz, so that, according to equation (5), Ω_0 = 45. From equation (9) we find that the half angle β = 2.8°. From equation (10) C_g = 0.055. Therefore c_g = 0.47 m/s. Fig. 5 indicates that the attenuation distance is about 1 km. Bearing in mind that cusp waves propagate at about 125° to the ship's track, attenuation due to water viscosity should not seriously affect the wake visibility unless the wake is longer than about 6 km. Fig. 1 confirms this.

Ignoring surface tension, on dimensional grounds or by a detailed calculation it can be shown that the e-folding length, r, of a narrow-V wake is given by:

$$r = \gamma U^3 / (\nu f_0^{\,2}) \quad (11)$$

where f_0 is the excitation frequency and γ is a constant that turns out to be approximately equal to $8.7\text{x}10^{-4}$. There may be other factors that reduce the wake length so that this can be regarded as an upper limit.

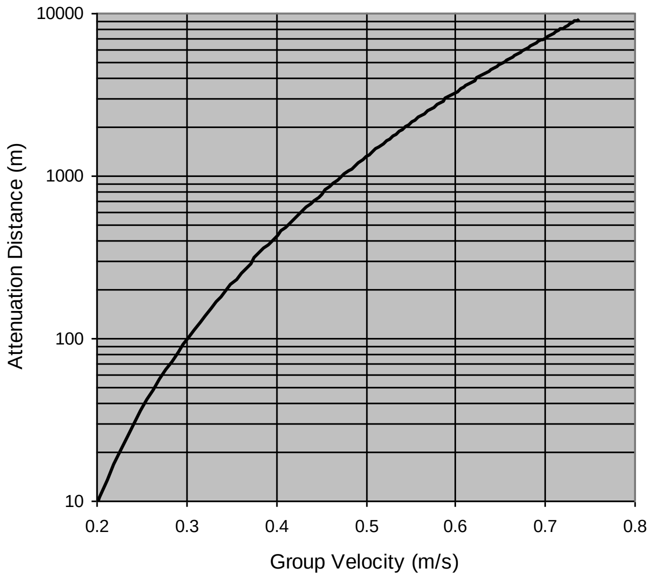

Fig. 5. The attenuation distance as a function of group velocity.

IV. Radar Imaging

At the frequencies of interest, radar and SAR returns in light winds seem to be adequately described by the Bragg scattering process. Indeed, Balser et al. have shown convincingly that Bragg scatter explains return from the narrow-V wake. Wright [11,12] has shown that the backscattered power is proportional to the power spectral density of surface height at the Bragg wavelength. There can also be second order effects from tilt and hydrodynamic modulations [13]. Wake imaging implies that the wake introduces changes in the water surface; these can be amplitude changes that affect the spectral density of the surface waves and surface velocity changes that create hydrodynamic modulation of capillary wave amplitudes. A wake theory, such as the IKW, that produces waves directly at the Bragg wavelength is attractive because there is a direct effect on radar scatter; unsteady wake waves from a propeller can only affect the radar return through second order perturbations.

The cusp waves of the propeller wake lie at the edge of the wake in a thin line. This is consistent with the radar imaging in that they are often only one resolution cell wide. Also they tend to be observed only when the ship is traveling close to the azimuthal direction so that velocity bunching leading to the randomization of image features [13] is not a factor.

An optical surface manifestation of the narrow-V wake is not a feature that has been identified by other workers. This may be because it is obscured; near the ship, narrow-V wakes are embedded within the turbulent wake and the cusp wavelength may be small. Moreover, for some distance behind a ship moving at service speeds, there is almost always a layer of foam over the sea surface. This is generated within the boundary layer and created by turbulence from the propeller. It would be useful to study the surface properties of unsteady wakes experimentally in a direct manner.

V. Conclusions

The IKW can be a useful model to explain the narrow-V wake at L-band and lower frequencies, depending on the incidence angle. Neither the Kelvin wake nor the IKW are viable models for C-band and X-band radars as has been recognized from early studies of Seasat wakes. This is because capillary waves at microwave wavelengths tend to be strongly damped. The model described here seems to be appropriate for microwave frequencies but the IKW will also be a candidate at frequencies at or lower than L-band. The new model is based on the unsteady wake and does not suffer from the attenuation problem. Attenuation is reduced because the cusp waves do not propagate perpendicularly to the ship track and their wavelength tends to be sufficiently high. The approach is founded on well-accepted theory and seems to explain the presence of narrow-V wakes with a range of opening angles appropriate to propeller action, though other sources are possible.

To confirm this, it will be necessary to estimate the amplitudes of cusp waves, to verify the radar scattering from the lines of cusps and to observe the surface perturbation directly. In the meantime, satellite imagery can be employed to correlate propeller blade frequency and ship speed with narrow-V wake angles.

If valid as expected, the theory described here can contribute to maritime surveillance by permitting estimates of propeller blade frequency and comparison with those from other sources; disparities can be used to trigger a closer look at the ship.

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